Oreshkin V. I.

Institute of High Current Electronics, SB RAS

Oreshkin E.V.

P.N. Lebedev Physical Institute of the RAS

Taking account of the radiation in a numerical simulation of an electrical explosion of conductorsds

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ABSTRACT

The mathematical models used to describe the processes involved in an electrical explosion of conductors are traditionally based on the magnetohydrodynamic approximation. To perform numerical calculations in this approximation requires preliminary knowledge of the equations of state of the conductor material in different phases for a wide range of thermodynamic parameters and corresponding transport coefficients. In solving problems related to the study of dense plasmas, it is of critical importance to characterize the plasma self-radiation in order to estimate the radiation losses and determine the spectral characteristics of the radiation for diagnostic purposes. There is a great variety of methods to characterize the self-radiation of plasmas. In this paper, we present a comparative analysis of different methods for calculating the characteristics of a dense plasma for which the conditions for thermodynamic equilibrium are satisfied.

INTRODUCTION

The process of electrical explosion of conductors (EECs) is traditionally simulated by using the magnetohydrodynamic (MHD) approximation. To perform numerical calculations in this approximation requires preliminary knowledge of the equations of state (EOSs) of the conductor material for a wide range of thermodynamic parameters and corresponding transport coefficients. The system of equations used to numerically simulate an EEC in terms of the MHD approximation generally consists of the equations of hydrodynamics that describe the laws of mass, momentum, and energy conservation; Maxwell's equations; Ohm's law; EOSs, which relate thermodynamic functions (pressure and internal energy) with thermodynamic parameters (temperature and density), and equations of radiation transfer. In solving problems related to the study of dense plasmas, it is of critical importance to characterize the plasma self-radiation in order to estimate the total radiation losses and determine the spectral characteristics of the radiation. The total radiation losses must be taken into account in an MHD simulation, and the spectral characteristics of radiation are used for plasma diagnostics. There is a wide variety of methods to describe the self-radiation of a plasma. Let us dwell on some of them.

RADIATION LOSSES FROM A DENSE PLASMAS

The radiation losses from a laboratory plasma, which increase with temperature, do not allow thermodynamic equilibrium to be established in the plasma. However, for dense and relatively low-temperature plasmas, the local thermodynamic equilibrium (LTE) approximation can be used. It is supposed that when LTE is established in a material, there is no equilibrium between the material and the emitted radiation, but the material itself is in equilibrium; that is, the distribution of ionized species and excited radiative levels in it can be described by the Saha–Boltzmann equation [1]. If the condition for the existence of LTE in a plasma is not fulfilled, to calculate the parameters of the plasma radiation requires the use of a nonequilibrium impact radiation model [2]. As the plasma of exploding conductors is quite dense and low-temperature, only models applied to LTE plasmas will be considered below.

For the case of a material being in thermodynamic equilibrium with the radiation it emits, the spectral distribution of the radiation density is described by the well-known relation (see, e.g., [1-4])

$$U_{\nu p} = \frac{8\pi h\nu^3}{c^3}\left(\exp\left(\frac{h\nu}{kT}\right)-1\right)^{-1} , \qquad (1)$$

where v is the radiation frequency, T is the temperature of the material, c is the velocity of light in vacuum, h is Plank’s constant, and k is Boltzmann’s constant.

The total equilibrium radiation density, which is obtained by integrating (1) over frequencies, is determined as $U_p = \frac{4\sigma_{SB}}{c}T^4$.

RADIATION LOSSES FROM A DENSE PLASMAS

In the case of LTE, the equation that describes the transfer of radiation of frequency v in a medium along a given direction has the form

$$\Omega \nabla I_v = k_v (I_{vp} - I_v) \quad , \qquad (2)$$

where I_v is the radiation intensity at the frequency v, $I_{vp} = \frac{c}{4\pi} U_{vp}$ is the equilibrium radiation intensity (Plank's function) [4], is unit vector of the direction of motion of photons, and kv is the absorption coefficient of the medium. In relation (2), the first term on the right describes the thermal source of photons at a given point of the emitting object and the second one describes the outflow of photons from the radiation field due to absorption.

Integration of (2) over angles yields an equation relating the radiation flux to the radiation energy density [1, 3]:

$$\nabla \mathbf{W}_v = c k_v \left(U_{vp} - c U_v \right) \quad , \qquad (3)$$

where $\mathbf{W}_v = \int I_v \Omega d\Omega$ is the radiation flux at the frequency v and $U_v = \frac{1}{c} \int I_v d\Omega$ is the radiation energy density at the same frequency. Equation (3) contains two independent variables; hence, to obtain a closed system, one more equation is needed which would relate these quantities.

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RADIATION LOSSES FROM A DENSE PLASMAS

For this purpose, both sides of equation (2) are multiplied by the vector Ω and integrated over angles and frequencies. After integration, a tensor associated with the radiation pressure tensor appears on the right. In the case of isotropic radiation, this new tensor is diagonal, and we obtain the desired equation of the form

$$\frac{c}{3k_\nu}\nabla U_\nu = -\mathbf{W}_\nu \quad . \qquad (4)$$

The multiplier $\frac{c}{3k_\nu}$ is an analog of the diffusion coefficient in atomic physics; therefore, the approximation based on equations (3) and (4) that describe radiation transfer is referred to as a diffusion approximation. The system of equations (3) and (4) makes it possible to calculate both the total radiation losses and the spectral characteristics of the radiation produced by an object.

The radiation transfer equations can be simplified by integrating (3) and (4) over frequencies, and then we have

$$\nabla \mathbf{W}_R = \frac{1}{\hat{L}}\left(4\sigma_{SB}T^4 - cU_R\right) \quad ; \qquad (3a)$$

$$\frac{c\hat{L}}{3}\nabla U_R = -\mathbf{W}_R \quad , \qquad (4a)$$

where $\mathbf{W}_R$ and U_R are the spectrum-integrated radiation flux and the energy density, respectively, and $\hat{L}$ is the mean free path of the photons. The system of equations (3a), (4a), in contrast to the system (3), (4), allows one to calculate only the total radiation losses.

RADIATION LOSSES FROM A DENSE PLASMAS

To calculate the mean free path of photons, various averaging methods are used [1, 3, 4]. However, for a dense plasma in which LTE conditions are fulfilled, the photon mean free path (Rosseland length) is most often used. It can be calculated by the following averaging formula:

$$L_R = \left(\int_0^\infty \frac{dU_{\nu p}}{dT} d\nu \right)^{-1} \cdot \int_0^\infty \frac{1}{k_\nu} \frac{dU_{\nu p}}{dT} d\nu \quad .$$

To calculate the Rosseland length, it is necessary to know the spectral dependence of the plasma absorption coefficient, which is determined by the charge state composition of the plasma and by the population of excited levels. These two characteristics of the plasma are found by solving the Saha–Boltzmann equations.

The system of radiation transfer equations can be simplified by the change of variable $U_R \to U_p$ in equation (4a), which then becomes

$$\mathbf{W}_R = -\frac{16\sigma_{SB} L_R}{3} \nabla T \quad . \qquad (5)$$

The multiplier $\frac{16\sigma_{SB} L_R}{3}$ in (5) is an analog of the thermal conductivity coefficient; therefore, the radiation

transfer described by equation (5) represents the radiant heat conduction approximation [1]. Being the simplest of all approximations, equation (5), like the system of equations (3a), (4a), makes it possible to calculate the total radiation losses.

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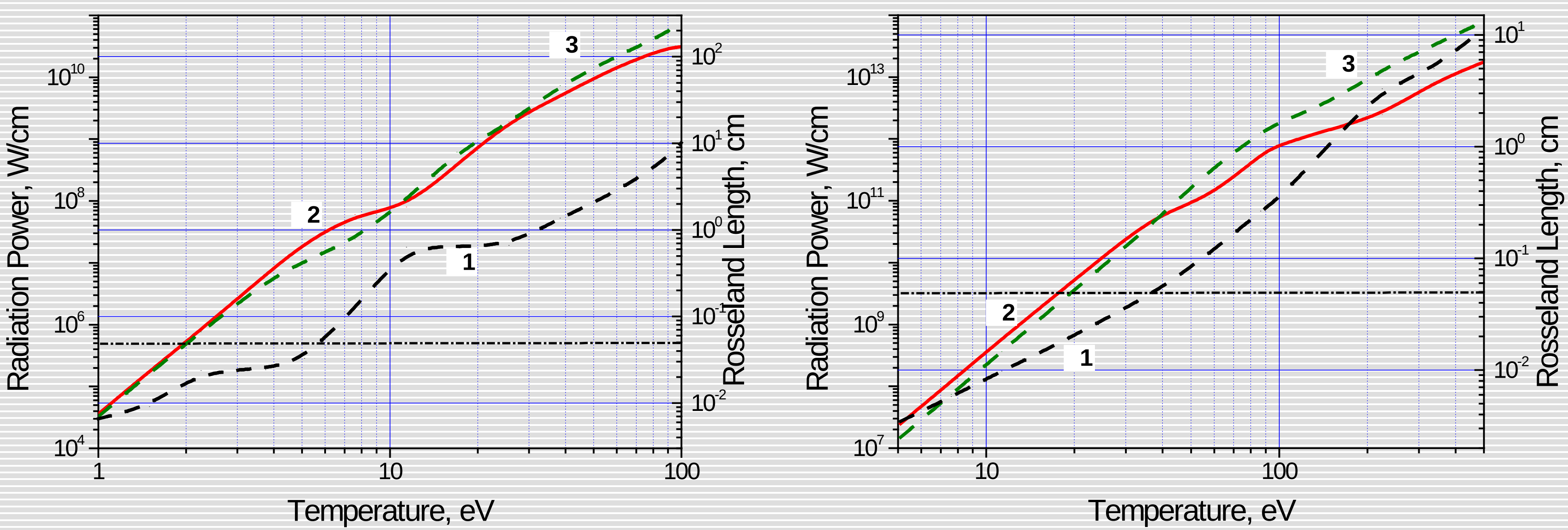
SIMULATION RESULTS

As the MHD simulation requires knowledge of the total radiation losses alone, one can restrict oneself to the system of equations (3a), (3b) or equation (5). The formal limit of applicability of these approximations is imposed by the condition that the characteristic size of the emitting region, R_w, is much smaller than the mean free path of a photon, i.e. $L_R << R_w$ (in our case, R_w is the radius of the plasma column).

Let us compare the radiation losses of a homogeneous plasma column calculated using (3a) and (4a) and using a nonequilibrium impact radiation model. The impact radiation model used in [6, 7] did not assume LTE; it included a system of balance equations for the populations of individual levels. The model took into account the following elementary processes: excitation of the plasma species by electron impact, spontaneous and induced radiative hyperfine transitions, impact ionization, photoionization, triple photorecombination, and dielectronic recombination. To find the radiation field, the spectral equations of radiation transfer along various directions were solved taking into account the effect of radiation reabsorption by spectral lines on the charge state composition of the plasma.

Comparing the results of calculations of the radiation losses by the two methods presented in Fig. 1, we see that they are almost identical in the domain of applicability of the approximation described by the system of equations (3a), (4a) ($L_R << R_w$). However, as can be seen from Fig. 1, a good match occurs even if the Rosseland lengths are equal to the characteristic size of the emitting region, and noticeable discrepancies are observed only when L_R is an order or two orders of magnitude greater than R_w.

SIMULATION RESULTS



a) b)

Fig. 1. Temperature dependence of the radiation characteristics of a plasma column of radius 0.05 cm for an aluminum plasma with an ion density of 1020 cm–3 (a) and for a copper plasma with an ion density of 1021 cm–3 (b). Curves 1, 2, and 3 represent, respectively, the Rosseland length, the radiation power calculated by formulas (3a-4a), and the radiation power calculated in terms of the nonequilibrium impact radiation model..

CONCLUSION

Thus, it can be concluded that the radiation transfer equations assuming LTE and not taking into account the dependence of the radiation intensity on frequency are valid not only in the domain of their formal applicability. They can also be used in the cases where the dimensions of the emitting region are comparable to the photon mean free path (Rosseland length).

REFERENCES

[1] Y. B. Zeldovich, and Y. P. Raizer, Physics of Shock Waves and High Temperature Hydrodynamic Phenomena Academic, 1966.

[2] V. Derzhiev, A. Zhidkov, and S. Yakovlenko, Radiation of ions in non-equilibrium dense plasma, Moscow: Energoatomizdat, 1986.

[3] B. Chetvertushkin, "Matematical Modeling in Problems of Emitting Gas Dynamics," Nauka, Moscow, 1985.

[4] D. Mihalas, “Stellar atmospheres,” San Francisco, WH Freeman and Co., 1978. 650 p., 1978.

[5] V. I. Oreshkin, S. Chaikovsky, N. A. Ratakhin, A. Grinenko, and Y. E. Krasik, ““Water bath” effect during the electrical underwater wire explosion,” Physics of Plasmas, vol. 14, no. 10, pp. 102703, 2007.

[6] V. Oreshkin, “Implosion of plasma liners in the presence of an axial magnetic field,” Russian Physics Journal, vol. 38, no. 12, pp. 1203-1210, 1995.

[7] V. Oreshkin, A. Artyomov, S. Chaikovsky, E. Oreshkin, and A. Rousskikh, “Simulation of the radiation from the hot spot of an X-pinch,” Physics of Plasmas, vol. 24, no. 1, pp. 012703, 2017.