Current in Planar Diode with a Moving Conducting Channel

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In this work, based on the current continuity equation and the Gauss theorem, we analyze the currents when the boundary of the conductive layer moves at a constant speed in the gap bounded by two parallel electrodes.

Relationship the diode current and voltage to the ionizationwave velocity in an axisymmetric statement

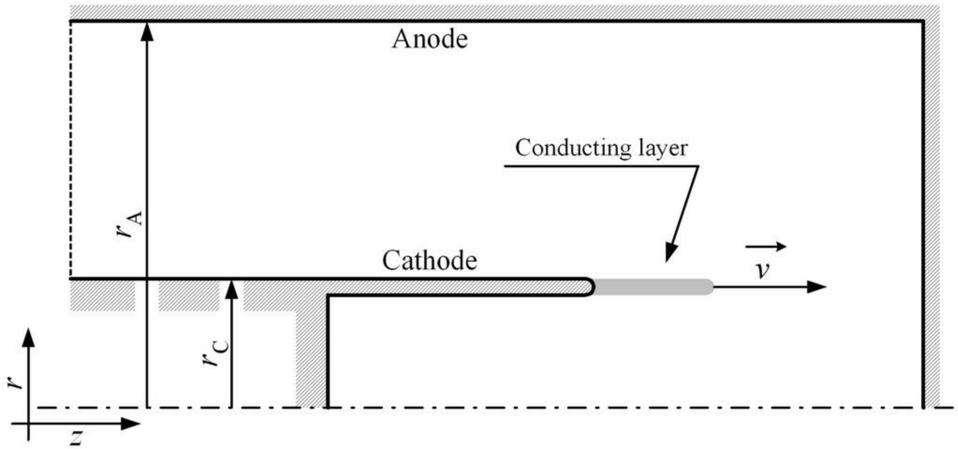

Fig. 1. Schematic of magnetically insulated coaxial diode with moving plasma channel.

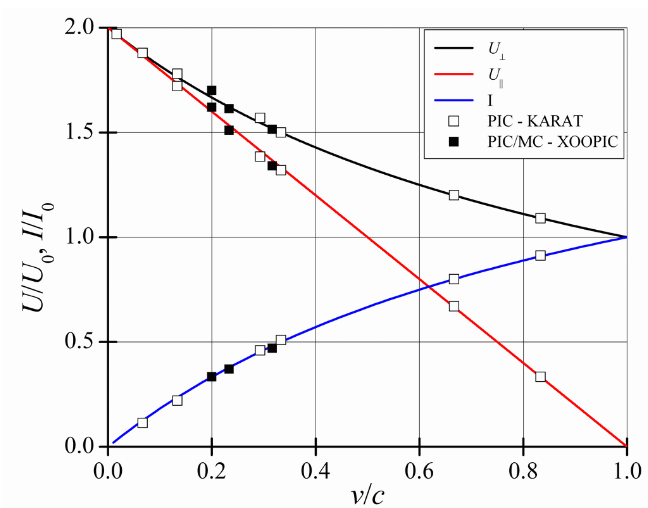

Fig. 2. Transverse and longitudinal diode voltages and diode current as functions of plasma channel velocity according to analytical model.

Previously was shown relates the diode current and voltage to the ionizationwave velocity from cathode to anode in an axisymmetric statement.

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$$R = \rho \frac{c}{v}$$

$$I = \frac{2I_0}{1 + \frac{c}{v}}$$

$$U_\perp = \frac{2U_0}{1 + \frac{v}{c}}$$

$$U_{||} = 2U_0 \cdot \left(1 - \frac{v}{c}\right)$$

Problem statement and Model description

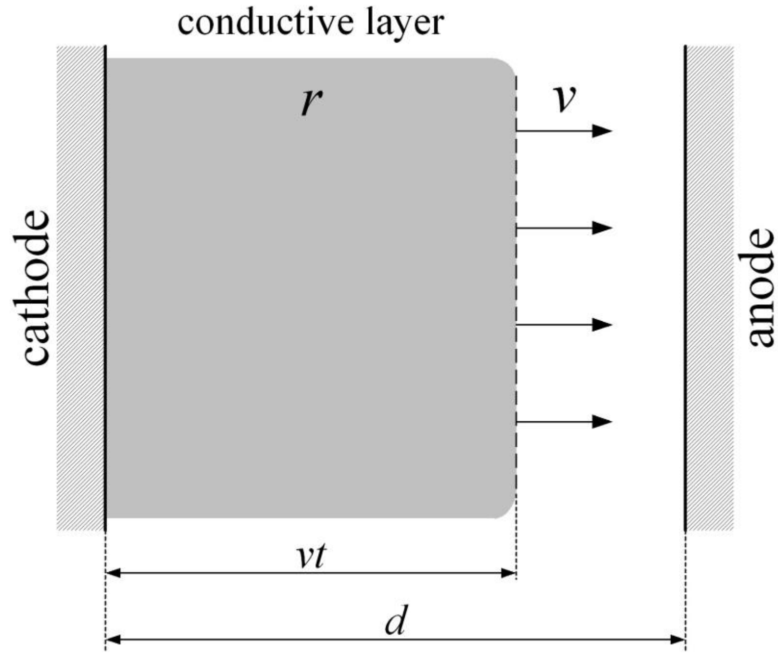

Fig. 3. Scheme of the problem statement with a moving conductive layer in a flat diode

- Let there be a gap bounded by two parallel electrodes
- Electrodes linear size is much larger than the distance between them
- The voltage between the electrodes is constant
- The boundary of the conductive layer starts from one of the electrodes
- Layer resistivity does not depend on time or coordinate
- The boundary of the conductive layer moves at a constant speed
- Emission from the cathode is able of sustain the full current of the diode

Let's use the continuity equation for current

$$div(j)+\varepsilon_0 div\left(\frac{\partial E}{\partial t}\right)=0 \qquad (1)$$

Under the conditions of the problem, only the z-component of the current and electric field can be considered

$$\frac{\partial}{\partial z}j+\varepsilon_0\frac{\partial}{\partial z}\frac{\partial E}{\partial t}=0 \qquad (1`)$$

Conduction current density through field strength and resistivity:

$$j=\frac{E}{r} \qquad (2)$$

The layer resistivity does not depend on time or coordinate.

$$\frac{1}{r}\frac{\partial E}{\partial z} + \varepsilon_0 \frac{\partial}{\partial t}\frac{\partial E}{\partial z} = 0 \qquad (3)$$

Let's use Gauss's law: $div(E) = \frac{1}{\varepsilon_0}\rho$ $\qquad \frac{\partial E}{\partial z} = \frac{1}{\varepsilon_0}\rho$ (4)

$$-\frac{1}{\varepsilon_0 r}\rho = \frac{\partial}{\partial t}\rho \qquad (5)$$

Equation solution (5) $\rho = \rho_0 e^{-\frac{t}{\tau}}$ where $\varepsilon_0 r = \tau$ (6)

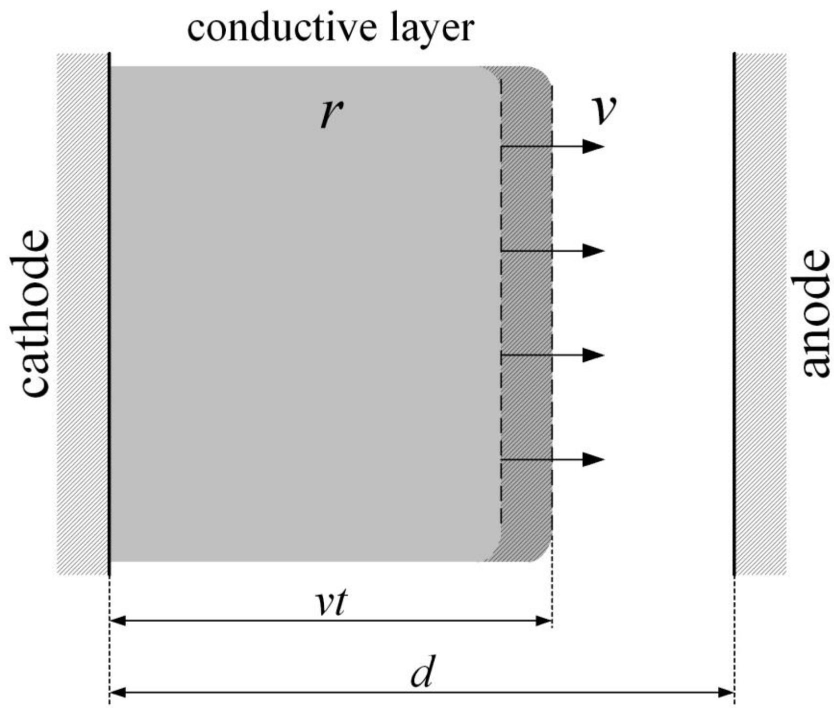

Fig. 4. "Accumulation" of conductivity ahead of the front of the conducting region.

In general, the boundary of the conductive layer moves with a velocity v. For each point in space inside the conducting layer there is its own value ρ_0 - the value of the space charge density at the moment when the boundary of the conducting layer was at this point. Then solution (6) can be rewritten as:

$$\rho(z) = \rho_0(z) e^{\frac{z - vt}{v\tau}} \qquad (7)$$

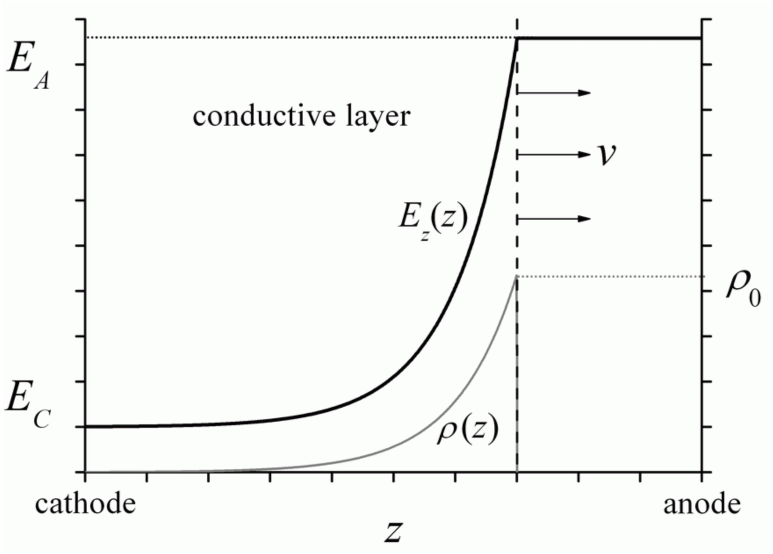

Fig. 5. Schematic distribution of the spacecharge density and electric field strength along the 0z axis.

At the cathode, the total current is the sum of the conduction current and the displacement current. At the anode, the total current is the displacement current.

$$\frac{E_C}{r} + \varepsilon_0 \frac{\partial E_C}{\partial t} = \varepsilon_0 \frac{\partial E_A}{\partial t} \qquad (8)$$

Let us integrate (4), but one should consider the integration constant

$$E(z) = \frac{1}{\varepsilon_0} \int_0^{z<vt} \rho(z) dz + E_C \qquad (9)$$

The voltage between the electrodes is constant

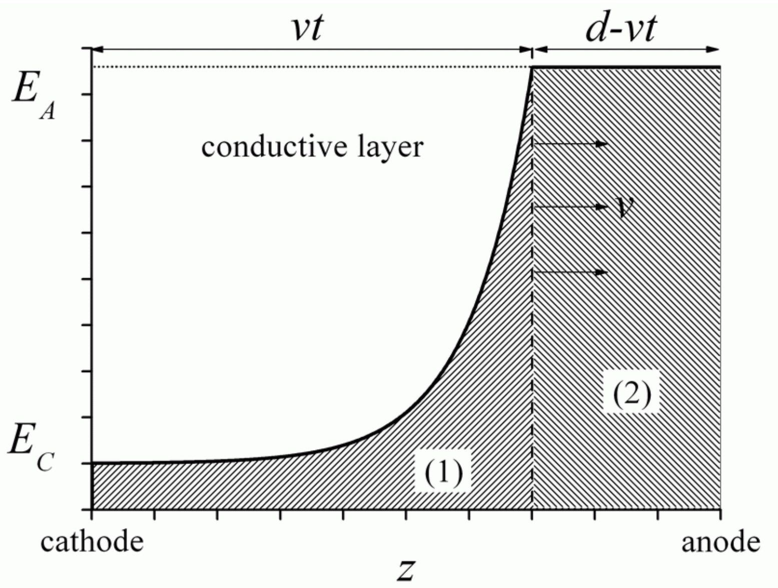

Fig. 6. Voltage as an integral of electric field strength

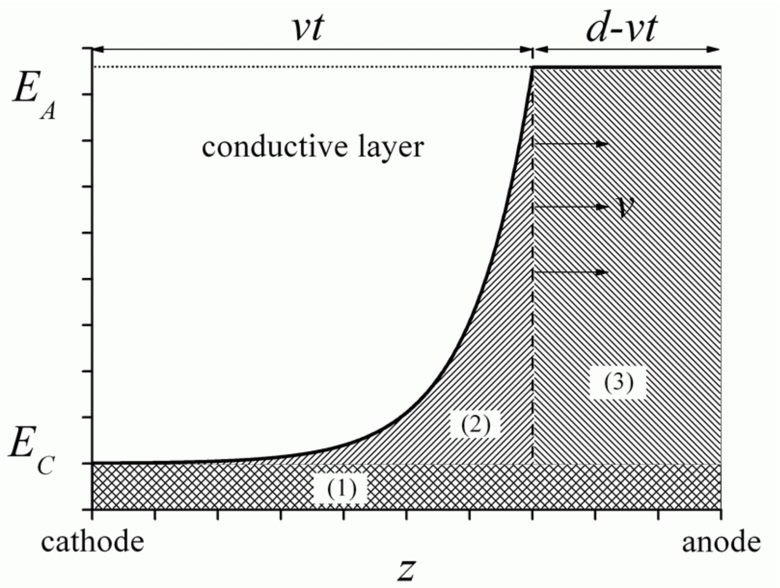

Fig. 7. Voltage as an integral of charge distribution with an integration constant E_c

Equation for Fig.6:

$$U = \underbrace{\int_0^{vt} E(z)dz}_{1} + \underbrace{(d - vt)E_A}_{2} \qquad (10)$$

Equation for Fig.7:

$$U = \underbrace{d \cdot E_C}_{1} + \underbrace{\frac{1}{\varepsilon_0}\int_0^{vt}\int_0^{z} \rho(z')dz'dz}_{2} + \underbrace{\frac{d - vt}{\varepsilon_0}\int_0^{vt} \rho(z)dz}_{3} \qquad (11)$$

(7): $\rho(z)=\rho_0(z)e^{\frac{z-vt}{v\tau}}$

(8): $$\frac{E_C}{r}+\varepsilon_0\frac{\partial E_C}{\partial t}=\varepsilon_0\frac{\partial E_A}{\partial t}$$

(9): $$E_A=\frac{1}{\varepsilon_0}\int_0^{vt}\rho(z)dz+E_C$$

(11): $$U=d\cdot E_C+\frac{1}{\varepsilon_0}\int_0^{vt}\int_0^{z}\rho(z')dz'dz+\frac{d-vt}{\varepsilon_0}\int_0^{vt}\rho(z)dz$$

$$x=\frac{z}{d}\qquad x_b=\frac{vt}{d}\qquad \lambda=\varepsilon_0\frac{vr}{d}\qquad a=\frac{E}{U/d}\qquad s=\rho\frac{d^2}{\varepsilon_0 U}\qquad (12)$$

$$\begin{cases}\dfrac{a_C}{\lambda}+\dfrac{\partial a_C}{\partial x_b}=\dfrac{\partial a_A}{\partial x_b}\\ a_A=a_C+\displaystyle\int_0^{x_b}s_0(x)\cdot\exp\left(\frac{x-x_b}{\lambda}\right)dx\\ 1=a_C+(1-x_b)\displaystyle\int_0^{x_b}s_0(x)\cdot\exp\left(\frac{x-x_b}{\lambda}\right)dx+\int_0^{x_b}\int_0^{x}s_0(x')\cdot\exp\left(\frac{x'-x_b}{\lambda}\right)dx'dx\end{cases}\qquad (13)$$

Reduced form of the system (13)

$$\frac{\partial a_A}{\partial x_b} = \frac{1}{\lambda}\left(1 - a_A \cdot \left(1 - x_b\right)\right) \qquad (14)$$

Starting and ending time points

$$x_b \to 0: \quad a_A = 1 \quad \frac{\partial a_A}{\partial x_b} \equiv 0$$

$$x_b \to 1: \quad \frac{\partial a_A}{\partial x_b} \equiv \frac{1}{\lambda}$$

$$\frac{\partial\left(\frac{E_A}{U/d}\right)}{\partial\left(\frac{vt}{d}\right)} = \frac{1}{\varepsilon_0 \frac{vr}{d}} \Rightarrow \quad \varepsilon_0 \frac{\partial E_A}{\partial t} = \frac{U}{rd}$$

At the moment of closure, when the conductive layer contact the anode, the total current density of the diode is determined by Ohm's law.
It is a elementary conclusion, but it can only be obtained using equation (7)

GENERAL SOLUTION

The general solution of equation (14) gives the dependence of the field a_m at the anode on time x_b:

$$a_A = e^{\frac{(x_b-1)^2}{2\lambda}}\left[\sqrt{\frac{\pi}{2\lambda}}\cdot erf\left(\frac{(x_b-1)}{\sqrt{2\lambda}}\right)+e^{-\frac{1}{2\lambda}}+\sqrt{\frac{\pi}{2\lambda}}\cdot erf\left(\frac{1}{\sqrt{2\lambda}}\right)\right] \qquad (15)$$

Dependence of the total diode current on time::

$$\frac{\partial a_A}{\partial x_b} = \frac{1}{\lambda}+\left[\sqrt{\frac{\pi}{2\lambda}}erf\left(\frac{x_b-1}{\sqrt{2\lambda}}\right)+e^{-\frac{1}{2\lambda}}+\sqrt{\frac{\pi}{2\lambda}}erf\left(\frac{1}{\sqrt{2\lambda}}\right)\right]\cdot\left(\frac{x_b-1}{\lambda}\right)\cdot e^{\frac{(1-x_b)^2}{2\lambda}} \qquad (16)$$

Relationship between the value of the electric field strength at the anode and the total diode current:

$$J = \frac{U}{d\cdot r}-\frac{E_A}{r}\cdot\frac{(d-vt)}{d} \qquad E_A = \frac{(U-J\cdot d\cdot r)}{(d-vt)} \qquad (17)$$

$$\lambda = \varepsilon_0 \frac{vr}{d}$$

$$a = \frac{E}{U/d}$$

$$x_b = \frac{vt}{d}$$

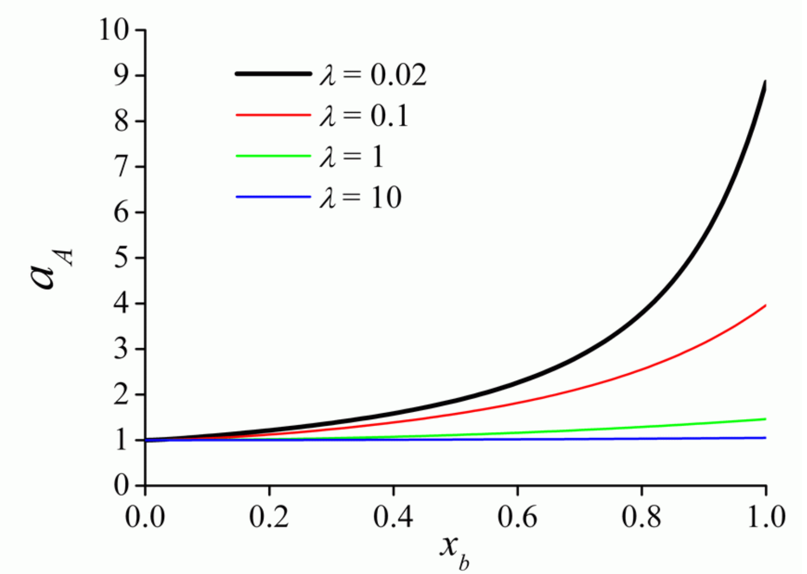

Fig. 8. Time dependence of the electric field strength near the anode for various λ values.

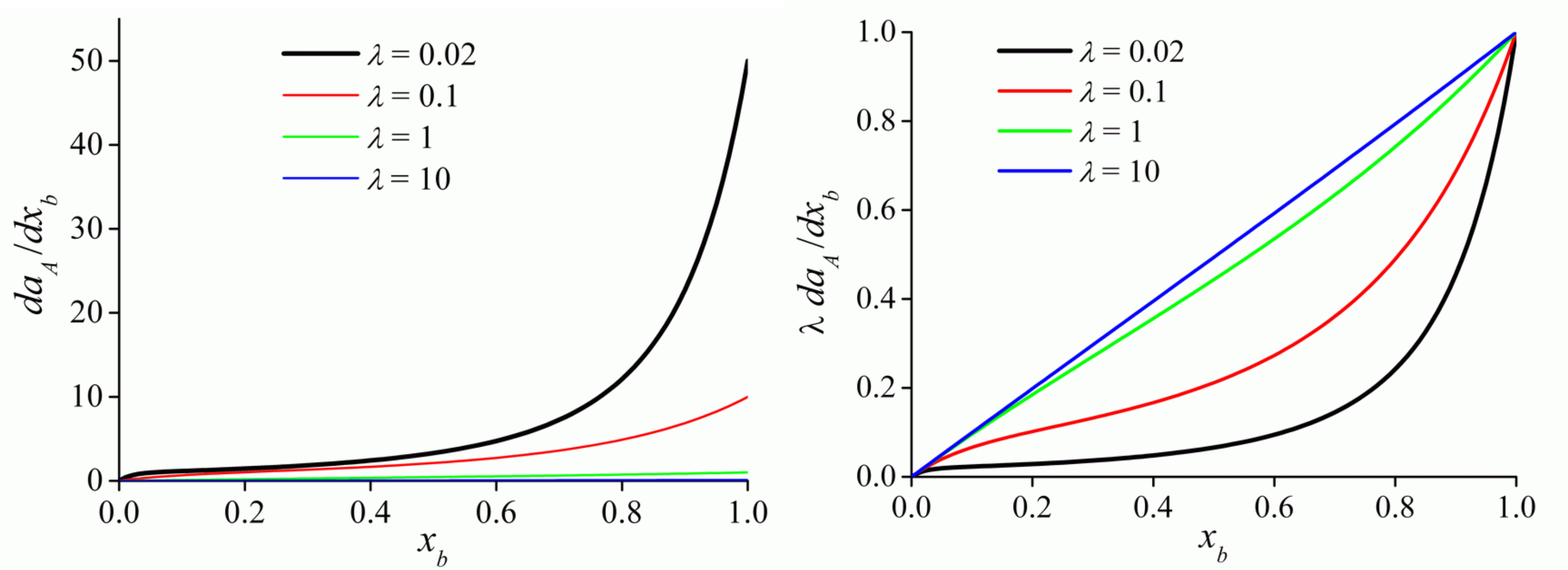

Fig. 9. Time dependence of the total diode current density for various λ values.

$$\lambda = \varepsilon_0 \frac{vr}{d} \quad a = \frac{E}{U/d} \quad s = \rho \frac{d^2}{\varepsilon_0 U}$$

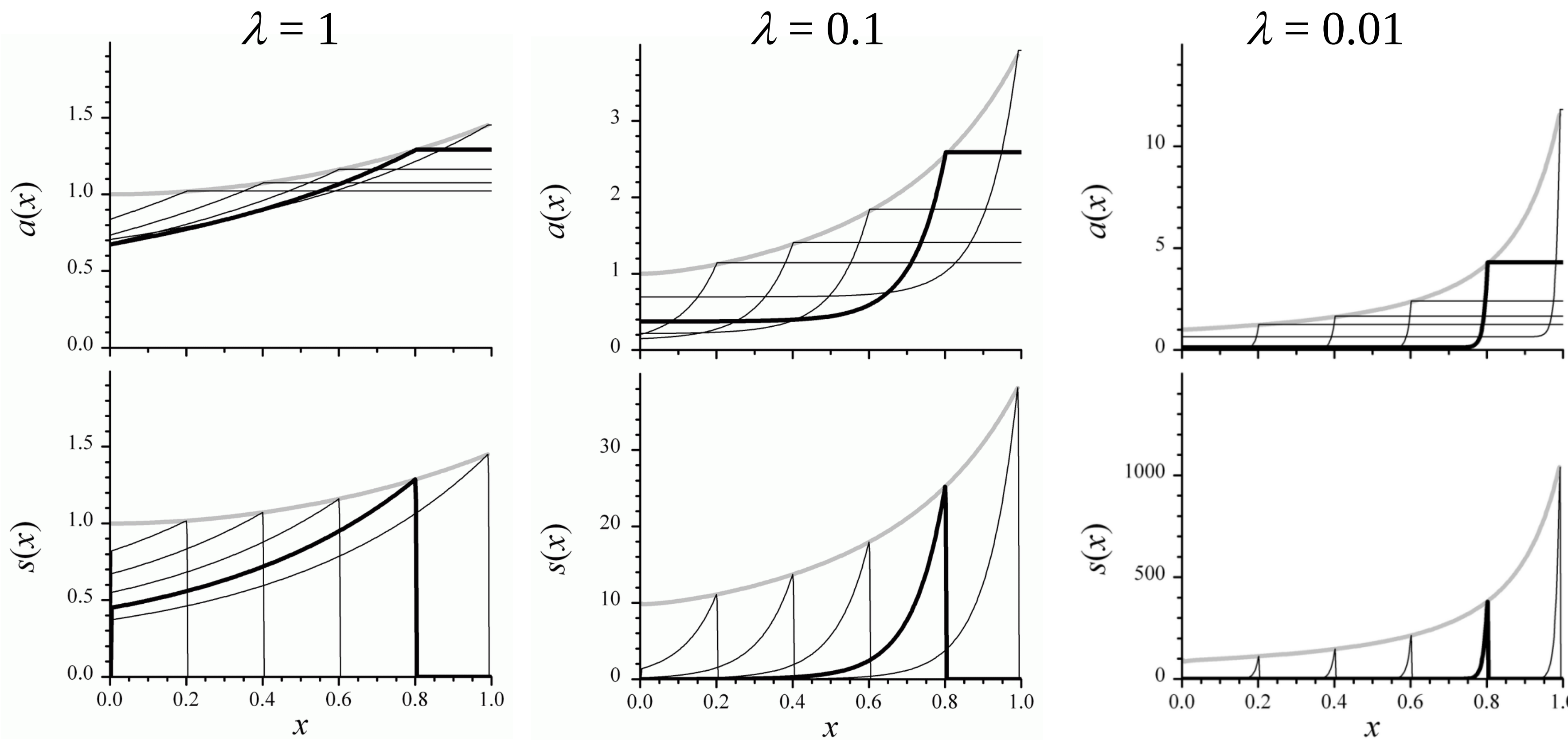

Fig. 10. Spacecharge distribution and electric field strength in the diode at different times for various λ values.

Conclusions

- An analytical model of the development of a conducting channel with a constant resistivity in the plane case is described.
- It is shown that the electric field strength decreases monotonically into the depth of the conducting layer. It is associated with the presence of an uncompensated space charge inside it.
- This space charge appears due to the motion of the boundary of the conductive layer and its finite resistivity.
- It is shown that at the moment of closure, when the conductive layer contact the anode, the total current density of the diode is determined by Ohm's law.
- The described theory was tested using the phenomenological model of the motion of the conducting layer using the KARAT code

$$\frac{\partial a_A}{\partial x_b} = \frac{1}{\lambda}\left(1 - a_A \cdot \left(1 - x_b\right)\right)$$